1.0 Introduction

The objective of this note is to document the methods used as the first step in estimating pressure drop in the coolant return line of the Pixel Detector. The analysis is for the evaporative cooling system using C_4F_{10} coolant. The objective of this first step is to estimate the pressure anticipated at the compressor inlet, in terms of the return line sizes being considered for the ATLAS Pixel Detector. This objective is first approached assuming a homogeneous flow condition in the two-phase, flow region of the Pixel Detector stave. More rigorous methods can be used to treat a separated flow condition between phases, and this approach may be justified if a more exact model of flow conditions in the evaporated region is needed. For the present, we confine our study to providing insight into the return line size.

The cooling system employs an evaporative cooling concept to extract heat from the pixel modules. A fluorinert liquid, in this case C_4F_{10}, is throttled through a small diameter injector. The two-phase, liquid-vapor mixture passes through a small diameter passage, which accepts heat from an array of pixel modules. In this analysis, we choose the stave in the barrel region to study. The stave orientation is horizontal, which simplifies determination of the pressure gradient in this mixed flow regime.

Tests have been successfully performed on several staves in parallel, results of which were discussed at the LHC Review [LHC Review of the Pixel TDR, July 7, 1998, Greg Hallewell]. This information is used to guide the estimation of the fluid properties at the stave exit. In this test series the heat transferred to the evaporated region was believed to be 72 W at a flow of 0.7 cm^3/sec. This particular data point corresponded to an inlet of 1.965 bar, and a stave exit pressure of 0.45 bar. Stave surface temperatures varied from –11 at the entrance to –14°C at the exit. An opportunity will soon exist to improve our understanding of the thermal/hydraulic performance, as plans call for further stave testing with more complete instrumentation.

The ATLAS Pixel Detector Collaboration has decided to combine the fluid exiting from two individual staves into one flow circuit, referred to a modularity of two. The internal tube diameter specification presently under consideration for this combined circuit is: 4.7 mm diameter for 1.5 m, 5.9 mm diameter for 5.4 m, and 9.4 mm diameter for the next 25 meters.

2.0 Status and Preliminary Conclusions

An estimate was made of the pressure drop through the stave and for the portion of the return line out to PPB3. The total path length comprises a distance of 31.9 meters. The specific flow parameters used in the analysis are summarized in Appendix B and C. Appendix B details a parameter set where the fluid leaving the stave has been completely evaporated. In this case the fluid is assumed to enter the injector at +15°C, and after throttling enters the heated region at a quality of 32%. For this state, the mass flow of 0.7 cm^3/sec will completely vaporize in extracting the 72 W. Appendix C in contrast details the flow parameter set assuming the fluid enters the injector sub-cooled to –5 °C. This condition leads to a quality downstream of the injector of 10% and an exit quality of 80%. Theoretically, it would take another 23 W to evaporate the remaining fluid.

Thus far the pressure loss in the return line has been estimated only for the case where the fluid media leaves the stave dry. This approach is temporarily justified by our interest of keeping this initial appraisal simple and timely. We anticipate that the pressure drop for two-phase flow in the return line will be considerably higher under certain conditions. If we can not reach a satisfactory solution with the most optimistic case, it seems hardly worth the effort to study the two-phase flow case in detail. However, a simple two-phase model was developed for the stave to gain some appreciation of the pressure loss in the stave.

The solution in the return line is for isothermal flow, a state of thermal balance with the surroundings. If heat were removed the pressure drop would be reduced, and conversely for heat addition the pressure drop would increase. Means for purposely extracting heat from the out-going vapor conflicts with our desire to find the smallest package. For the present, adoption of counterflow tubing, i.e., exchanging heat with the incoming fluid to accomplish this objective clouds the issues at hand.

The results of these preliminary calculations suggest the following:

- The pressure drop in the stave is a significant fraction of the total pressure available if the flow enters at a quality of 32% and exits dry. The pressure drop was calculated for an effective diameter of 3.4[1] mm, and found to be 123 mbar, of which 108 mbar is due to friction.
- Pressure drop in the stave at a lower entering quality is lower, but then the pressure drop in the return line may be higher offsetting this initial effect.

As evidenced *by this specific set of stave test data*[2], only 450 mbar at the stave exit exists for providing fluid return. The calculated stave pressure drop assuming a 30% inlet quality, becomes a significant fraction of the available energy, 32%. Increasing the tube to 5-mm diameter for this design point would net approximately 120 mbar more for the return line sizing, and substantially improving the stave thermal performance as well.

The first flow solution for the return line was constrained to use the desired tube sizes i.e., 4.7-mm for the first 1.5 m, etc. These conditions lead to the available energy being quickly expended. To overcome this, different line diameters for the specified path lengths were arbitrarily used. The case chosen is:

- 5.9-mm(versus 4.7) for 1.5 m
- 5.9-mm(versus 5.9) for 5.4 m
- 13-mm(versus 9.4) for 25 m

For the indicated line sizes, the pressure at the end of 31.9 meters was 215 mbar, without accounting for elbow losses. At the stave exit section, the velocity is of the order of 19 to 30 m/sec, for exit qualities of 1(gas) and 0.8 (mixture) respectively. As an example, an elbow loss to couple the two flows could be of the order of 30 mbar.

A recommendation of the appropriate tube diameters for the return flow circuit requires some consideration of compressor performance. One is faced with the prospect of selecting several, if not many large compressors with sufficient pumping speed to handle the volume flow rate. At the particular pressure chosen, 215 mbar, the compressors must handle nominally 190 m^3/hr throughput for the barrel region alone. As a matter of comparison, one Edwards compressor can handle roughly 6 m^3/hr at 250 mbar.

[1] Estimate from the Genoa stave profile

[2] Higher exit pressures have been tested. The thermal performance of the stave influences this choice. The higher the thermal performance, the higher permissible exit pressure.

Additional information is needed before one can be confident the appropriate line sizes have been selected. If one adopts the premise that some flexibility still exists for choosing slightly larger line sizes, it can be stated the C_4F_{10} fluid option still appears practical. Also, at this point it is not clear whether one should avoid having two-phase flow in the return line. It is suggested that one finish examining the return line size based on flowing dry vapor as outlined here. Concomitant with this choice, the inlet to the fluid injector the fluid is only slightly sub-cooled, and the entering fluid quality is high with possibly a bubbly flow appearance in the heated region.

2.0 Analysis

2.2.1 Pressure/Flow Analysis for Pixel Detector Stave

The following nomenclature will be used in the analysis of the pixel detector evaporative cooling system:

Fluid parameters

ρ , density, kg/m^3
μ, viscosity, Pa-s
k , thermal conductivity, W/m-k
c_p , specific heat, J/kg-K

Flow parameters

u , actual velocity, m/s
V , superficial velocity, m/s
G , mass flux, kg/s-m^2
x , quality of two phase mixture
α , void fraction in two phase flow

Subscripts

l , liquid
g , gas
h , homogeneous mixture
f , friction
lo , single phase, based on liquid mass flux
go , single phase, based on gas mass flux

Useful expressions

$$x = \frac{G_g}{G}, \quad AG_g = AGx = \rho_g u_g A_g = \rho_g u_g \alpha A, \qquad \alpha = \frac{1}{1+\left(\frac{u_g}{u_l}\frac{1-x}{x}\right)\left(\frac{\rho_g}{\rho_l}\right)},$$

$$V_g = \frac{G_g}{\rho_g}, \quad V_l = \frac{G_l}{\rho_l}, \qquad u_g = \frac{V_g}{\alpha}$$

If the flow is homogeneous $u_g = u_l$

$$\alpha_h = \frac{1}{1+\left(\frac{1-x}{x}\right)\left(\frac{\rho_g}{\rho_l}\right)}, \quad \text{and} \quad \rho_h = \rho_g \alpha_h + \rho_l (1-\alpha_h)$$

Substituting the homogeneous void fraction α_h into the expression for the homogeneous density ρ_h yields,

$\frac{1}{\rho_h} = \frac{x}{\rho_g} + \frac{1-x}{\rho_l}$, which is useful in our determination of pressure drop.

First, the pressure gradient in two-phase is composed of three terms, a frictional pressure gradient, a gravitational pressure gradient, and an acceleration pressure gradient. If it were not for a change in phase, a change in momentum would not occur.

The approach used to estimate the pressure gradients follows a procedure detailed in *"Boiling and Condensation and Gas-Liquid Flow"* by P. B. Whalley. This reference covers both homogeneous and separated flow. An estimate of the pressure drop in the heated region of the stave will assume homogeneous flow although later separated flow approach may be used if judged necessary.

Pressure gradient due to friction

$$\left(-\frac{dp}{dz}\right)_F = \frac{4\tau}{d} = \frac{4}{d}C_{fh}\frac{1}{2}\frac{G^2}{\rho_h} = \frac{4}{d}C_{flo}\frac{1}{2}\frac{G^2}{\rho_l}\left[\frac{C_{fh}\rho_l}{C_{flo}\rho_h}\right] = \frac{4}{d}C_{flo}\frac{1}{2}\frac{G^2}{\rho_l}\varphi_{lo}^2$$

The objective of the above expression is to transform the skin friction gradient into an expression based on the single-phase fluid properties and the mass flux of the two phase flow, and a two-phase flow multiplier, φ_{lo}^2. Whalley assumes that the homogeneous skin friction coefficient is approximately of the same magnitude as in the single phase, and this simplification results in:

$$\varphi_{lo}^2 = \frac{x\rho_l}{\rho_g} + 1 - x$$

Integrating over an inlet flow quality of x_1 to an exit flow quality of x_2 we obtain:

$$\left(-\Delta p\right)_F = \frac{2L}{d} C_{flo} \frac{G^2}{\rho_l}\left[1+\frac{X_0+X_i}{2}\right] \qquad (1)$$

where $X_0 = x_0\left[\frac{\rho_l-\rho_g}{\rho_g}\right]$, and $X_i = x_i\left[\frac{\rho_l-\rho_g}{\rho_g}\right]$

Terms x_0 and x_i denote the exit and inlet quality of the two phase flow. To calculate the frictional loss in the stave we need only know liquid fluid properties and the two flow quality terms. The Reynolds number will be based on the total mass flux, and the skin friction coefficient correlation with Reynolds number suggested by Whalley. There are other correlations for the skin friction coefficient that possibly should be considered.

$$R_n = \frac{Gd}{\mu_l}, \text{ and } C_{flo} = 0.079 R_n^{-0.25}$$

The expressions for pressure gradient induced by a gravitational change and the momentum change from the phase change can be integrated over the change in quality from inlet to exit. For these expressions we get:

Pressure change due to gravitational effects

$$\left(-\Delta p\right)_g = \frac{\rho_l g L \sin\theta}{\left[X_0 - X_i\right]}\left[\ln\left(\frac{1+X_0}{1+X_i}\right)\right] \qquad (2)$$

Pressure change due to momentum change

$$\left(-\Delta p\right)_a = \frac{G^2}{\rho_l}\left[X_0 - X_i\right] \qquad (3)$$

Referring to equation (1) we note that the friction loss is increased by the magnitude of the incoming fluid quality. If both qualities were zero the pressure loss would be limited to the skin friction resistance for the liquid. However, no evaporation would take place. It appears most desirable to reduce the injected mass flow to produce an exit quality of 0.99 (~complete evaporation), and to sub-cool the incoming liquid to produce an entrance quality of <0.1. The vapor exiting the stave would be essentially dry, and this condition would present fewer flow problems in the vapor return lines.

Appendix A and D provide example calculations for differing fluid quality, inlet and outlet. Some speculation of fluid parameters was used to make these predictions, which can be adjusted when more information becomes available. Also, it is possible to examine the benefit of using a more rigorous form for the two-phase flow model, such as separated flow in place of the homogeneous flow assumption.

2.2.2 Exit Line Pressure Loss

An estimate is made of the pressure gradient in the return lines to the compressor assuming that the fluid is entirely C_4F_{10} vapor, with initial properties corresponding to the stave exit conditions. Because of space constraints in the inner detector region, it becomes necessary to use a small tube diameter initially, finally increasing to a larger diameter in less constraining areas. Our objective is to estimate the tube diameter and associated path length scenario that will lead to a reasonable compressor inlet pressure.

For the first estimation, we assume the flow in the return line to be isothermal. The thermal boundary options are numerous:

- isothermal
- adiabatic
- cooled, and or
- heated

The three choices, other than isothermal, in the confining space immediately downstream of the stave would require additional space to effect the heat exchange. An uninsulated aluminum line in the sub-cooled region of the inner detector would probably satisfy the isothermal boundary condition. In the outer regions were the surrounding temperatures are approaching room temperature this condition may not be satisfied. Our approach to handling the pipe thermal boundary will change, as more information becomes available.

Thermal energy exchange with the surroundings, in and out of the pipe, and the internal heating effects must balance for the isothermal boundary condition to be satisfied. For this state the fluid temperature remains constant throughout, and the thermal energy flowing into the tube is approximately equal to the increase in kinetic energy of the gas. Isothermality is not uncommon in naturally uninsulated pipes, where velocities are low (sub-sonic), and the temperatures inside and outside the pipe are nearly the same. The differential equation for the isothermal compressible flow problem is given by:

$\frac{pdp}{G^2RT} + \frac{dV}{V} + \frac{4f}{2d}dl = 0$, where f[3] is constant over the path l, since Reynolds number is similarly constant[4]. Integrating the differential equation leads to:

$$p_1^2 - p_2^2 = G^2RT\left[2\ln\left(\frac{p_1}{p_2}\right) + 4f\frac{l}{d}\right] \tag{4}$$

[3] f is equal to $0.079R_n^{-0.25}$

[4] $R = \frac{Vd\rho}{\mu}$, V increases in direct proportion to a decrease in ρ, and μ is constant because temperature is constant. Hence, Reynolds number and the skin friction coefficient are constant.

An explicit relationship does not exist for p_2. However, in those situations where we show that $2\ln\left(\frac{p_1}{p_2}\right)$ is small in comparison to $f\,\frac{l}{d}$ we can obtain p_2 directly.

For isothermal flow in long ducts of constant diameter there is a physical limiting duct length that can not be exceeded. In sub-sonic flow, for each case of a higher inlet duct Mach number the duct length must decrease. As the inlet Mach number approaches the critical Mach number, which is slightly less than Mach 1, the duct length becomes quite short. The limit of course is for a duct length of zero where the inlet Mach number is the critical Mach. Number. The relationship[5] that defines the duct length limit is given by:

$$\frac{M_1^2}{M_2^2} = 1 - kM_1^2\left[2\ln\left(\frac{M_1}{M_2}\right) + 4f\,\frac{l}{d}\right], \qquad (5)$$

where $M_2 = \left[\frac{1}{k}\right]^{1/2}$, this expression reduces to:

$4k\,\frac{L_{\max}}{d} = \frac{1-kM^2}{kM^2} + \ln kM^2$, when one inserts the expression for the critical Mach number M_2 into *(5)*.

Appendix A provides a few examples for the pressure loss in the exit tube, where the output of two staves has been combined into one tube. The results indicate that if one uses a 4.7-mm diameter tube initially that the flow becomes critical within 2.5 meters. The path length for this tube diameter was chosen to be 1.5 m. This choice results in an excessive pressure loss in the first 1.5 meters. It is recommended that the first tube diameter be at least 5.9 mm.

[5] References for equation 4 and 5 were extracted from *Elementary Fluid Mechanics*, Vennard & Street, 6th edition, and *The Dynamics and thermodynamics of Compressible Flow* by Ascher Shapiro.

System flow analysis-Stave Barrel C_4F_{10}, +15.8 Deg. C inlet to injector, Quality of 32% after injection

1.0 Pressure Drop Calculations In Stave

Frictional Pressure Drop-Equation 1

$\nu_{liq} := 0.405 \cdot 10^{-6} \frac{m^2}{s}$ at -15 deg C, after injection

$\rho_{liq} := 1653.4 \cdot \frac{kg}{m^3}$ $\rho_g := 6.68 \cdot \frac{kg}{m^3}$ $x_i := 0.32$ $x_o := 0.999$ $\mu_{liq} := \rho_{liq} \cdot \nu_{liq}$ $\mu_{liq} = 6.696 \cdot 10^{-4} \cdot Pa \cdot s$

$mdot := 1.1 \cdot 10^{-3} \frac{kg}{s}$ $d := 0.0034 \cdot m$ $A := \frac{\pi}{4} \cdot d^2$ $A = 9.079 \cdot 10^{-6}\ m^2$

$G := \frac{mdot}{A}$ $G = 121.156 \frac{kg}{m^2 \cdot s}$ $G^2 = 1.468 \cdot 10^4 \frac{kg^2}{m^4 \cdot s^2}$

$R_{liq} := \frac{G \cdot d}{\mu_{liq}}$ $R_{liq} = 615.164$ $X_o := x_o \cdot \left(\frac{\rho_{liq} - \rho_g}{\rho_g} \right)$ $X_i := x_i \cdot \left(\frac{\rho_{liq} - \rho_g}{\rho_g} \right)$

$C_{flo} := 0.079 \cdot R_{liq}^{-0.25}$ $C_{flo} = 0.016$ $L := 0.8\ m$

$\Delta P_F := \frac{2 \cdot L}{d} \cdot C_{flo} \cdot \frac{G^2}{\rho_{liq}} \cdot \left(1 + \frac{X_o + X_i}{2} \right)$ $\Delta P_F = 1.084 \cdot 10^4\ Pa$ $\Delta P_F = 81.311 \cdot torr$

$\Delta Pmbar_F := \frac{\Delta P_F \cdot 1000}{1 \cdot 10^5\ Pa}$ $\Delta Pmbar_F = 108.406$

Gravitational Pressure Drop

This pressure drop contribution is zero because the stave position remains horizontal

Accelerational Pressure Drop-Equation 3

$\Delta P_A := \frac{G^2}{\rho_{liq}} \left(X_o - X_i \right)$ $\Delta P_A = 1.486 \cdot 10^3\ Pa$ $\Delta Pmbar_A := \frac{\Delta P_A \cdot 1000}{1 \cdot 10^5\ Pa}$ $\Delta Pmbar_A = 14.86$

Combined Pressure Drop

$\Delta P := \Delta P_F + \Delta P_A$ $\Delta P = 1.233 \cdot 10^4\ Pa$ $\Delta Pmbar := \Delta Pmbar_F + \Delta Pmbar_A$ $\Delta Pmbar = 123.266$

Since the pressure decays this amount we can expect the exit temperature to fall by ~5.5 Deg. C

Flow State Considerations

$T := -17$ gas viscosity temperature in degree C Expression for viscosity from Greg

$\mu_g := 1.1165 \cdot 10^{-5}\,Pa \cdot s + \left(4.0254 \cdot 10^{-8}\,T\right)\,Pa \cdot s$ $\mu_g = 1.048 \cdot 10^{-5}\,\frac{kg}{m \cdot s}$ $x_i = 0.32$

$\frac{\mu_g}{\mu_{liq}} = 0.016$ $\frac{\rho_g}{\rho_{liq}} = 4.04 \cdot 10^{-3}$ $x_o := 1$ set equal to a value slightly less than 1 to avoid divsion problems.

Void fraction at entrance and exit of stave

$G_{gin} := x_i \cdot G$ $V_{gin} := \frac{G_{gin}}{\rho_g}$ $V_{gin} = 5.804\,\frac{m}{s}$ $G_{gout} := x_o \cdot G$ $V_{gout} := \frac{G_{gout}}{\rho_g}$ $V_{gout} = 18.137\,\frac{m}{s}$

$G_{lin} := \left(1 - x_i\right) \cdot G$ $V_{lin} := \frac{G_{lin}}{\rho_{liq}}$ $V_{lin} = 0.05\,\frac{m}{s}$ $G_{lout} := \left(1 - x_o\right) \cdot G$ $V_{lout} := \frac{G_{lout}}{\rho_{liq}}$ $V_{lout} = 0\,\frac{m}{s}$

$$\alpha_i := \frac{1}{1 + \left(\frac{V_{gin}}{V_{lin}} \cdot \frac{1 - x_i}{x_i} \cdot \frac{\rho_g}{\rho_{liq}}\right)} \qquad \alpha_o := \frac{1}{1 + \left(\frac{V_{gout}}{V_{lout}} \cdot \frac{1 - x_o}{x_o} \cdot \frac{\rho_g}{\rho_{liq}}\right)}$$

must be based on actual velocities, here we are using superficial velocities

$\alpha_i = 0.5$ first estimate $\alpha_o := 1$ set equal to one since quality ratio is 1

$u_{gin} := \frac{V_{gin}}{\alpha_i}$ $u_{gin} = 11.608\,\frac{m}{s}$ $u_{lin} := \frac{V_{lin}}{1 - \alpha_i}$ $u_{lin} = 0.1\,\frac{m}{s}$ first cut at actual velocities

$$\alpha_{icorr} := \frac{1}{1 + \left(\frac{u_{gin}}{u_{lin}} \cdot \frac{1 - x_i}{x_i} \cdot \frac{\rho_g}{\rho_{liq}}\right)}$$

$\alpha_{icorr} = 0.5$ void fraction at inlet did not change, based on first cut at actual velocities

$V_{lout} = 0\,\frac{m}{s}$ liquid velocity at exit is really zero since quality is 1

$u_{gout} := \frac{V_{gout}}{\alpha_o}$ $u_{gout} = 18.137\,\frac{m}{s}$ $u_{lout} := \frac{V_{lout}}{1 - \alpha_o}$ $u_{lout} := 0\,\frac{m}{sec}$ expression becomes indeterminate, thus it is set equal to zero

$$\alpha_{ocorr} := \frac{1}{1 + \left(\frac{u_{gout}}{u_{lout}} \cdot \frac{1 - x_o}{x_o} \cdot \frac{\rho_g}{\rho_{liq}}\right)}$$

$\alpha_{ocorr} := 1$ void fraction at exit equal 1 since it is all vapor, and (1-xo/ xo) is zero.

$\alpha_{hin} := \dfrac{1}{1 + \dfrac{1 - x_i}{x_i} \cdot \dfrac{\rho_g}{\rho_{liq}}}$ $\alpha_{hin} = 0.991$ $\alpha_{hout} := \dfrac{1}{1 + \dfrac{1 - x_o}{x_o} \cdot \dfrac{\rho_g}{\rho_{liq}}}$ $\alpha_{hout} = 1$ homogeneous void fraction is noticeably different, at the inlet (exit of injector) since we assumed the velocities of the two phases are equal.

Flow Parameters at entrance and exit of stave

$\rho_{air} := 1.23 \dfrac{kg}{m^3}$ $\rho_{water} := 1000 \dfrac{kg}{m^3}$ $\psi := \left(\dfrac{\rho_g}{\rho_{air}} \cdot \dfrac{\rho_{liq}}{\rho_{water}} \right)$

$\dfrac{G_{gin}}{\psi} = 4.318 \dfrac{kg}{m^2 \cdot s}$ $\psi \cdot G_{lin} = 739.781 \dfrac{kg}{m^2 \cdot s}$ $\dfrac{G_{gout}}{\psi} = 13.493 \dfrac{kg}{m^2 \cdot s}$ $\psi \cdot G_{lout} = 0 \dfrac{kg}{m^2 \cdot s}$

slug flow in gas only leaving

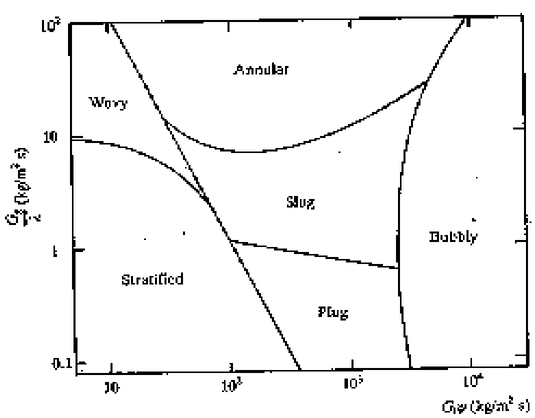


Fig. 2.6. Baker flow pattern map for horizontal flow in a tube.

2.0 Pressure Drop In Return Line (4.7 mm for 1.5 m causes excessive pressure drop)

Calculation for first 1.5 meters at 5.9 mm diameter, two staves feeding one return line $R_{universal} := 8313 \dfrac{J}{kg \cdot mol \cdot K}$

$M_w := 238 \dfrac{g}{g \cdot mol}$ $R \equiv \dfrac{R_{universal}}{M_w}$ $R = 34.929 \cdot \dfrac{J}{kg \cdot K}$ for C_4F_{10} universal gas constant

$T_r := (273 - 20.5)\ K$ $T_r = 252.5\ K$ $T := -20.5$ $G = 121.156 \dfrac{kg}{m^2 \cdot s}$

$\mu_g := 1.1165 \cdot 10^{-5}\ Pa \cdot s + \left(4.0254 \cdot 10^{-8}\ T\right)\ Pa \cdot s$ $\mu_g = 1.034 \cdot 10^{-5} \dfrac{kg}{m \cdot s}$

$d_{12} := .0059\ m$ $\quad A_{12} := \frac{\pi}{4} \cdot d_{12}^{\,2}$ $\quad A_{12} = 2.734 \cdot 10^{-5}\ m^2$ $\quad G_{12} := \frac{mdot \cdot 2}{A_{12}}$ $\quad L_{12} := 1.5\ m$

$G_{12} = 80.469\ \frac{kg}{m^2 \cdot s}$

$R_{12} := \frac{G_{12} \cdot d_{12}}{\mu_g}$ $\quad R_{12} = 4.592 \cdot 10^4$ $\quad f_{12} := 0.079 \cdot R_{12}^{\,-0.25}$ $\quad f_{12} = 5.397 \cdot 10^{-3}$ $\quad p_{sexit} := 0.45 \cdot 1 \cdot 10^5\ Pa$

$p_2 := \left[p_{sexit}^{\,2} - G_{12}^{\,2} \cdot R \cdot T_r \cdot \left(\frac{4 \cdot f_{12} \cdot L_{12}}{d_{12}} \right) \right]^{\frac{1}{2}}$ $\quad p_2 = 4.137 \cdot 10^4\ Pa$ $\quad p_{2mbar} := \frac{p_2}{1 \cdot 10^5\ Pa} \cdot 1000$ $\quad p_{2mbar} = 413.712$

$\Delta P := p_{sexit} - p_2$ $\quad \Delta P = 3.629 \cdot 10^3\ Pa$

compare 4fL/d versus 2ln(p1/p2)

$2 \cdot \ln\left(\frac{p_{sexit}}{p_2} \right) = 0.168$ $\quad \frac{4 \cdot f_{12} \cdot L_{12}}{d_{12}} = 5.488$ approximation is fairly reasonable

$\rho_2 := \frac{p_2}{R \cdot T_r}$ $\quad \rho_2 = 4.691\ \frac{kg}{m^3}$ $\quad \rho_{sexit} := \frac{p_{sexit}}{R \cdot T_r}$ $\quad \rho_{sexit} = 5.102\ \frac{kg}{m^3}$ versus 5.14 kg/m^3 from 3M table , within 0.7% by using universal gas constant

$Q_2 := \frac{mdot \cdot 2}{\rho_2}$ $\quad Q_2 = 1.688 \circ \frac{m^3}{hr}$ at this point one Edwards compressor would handle 5.9 stave groups(11.8 staves)

$u_2 := \frac{Q_2}{A_{12}}$ $\quad u_2 = 17.154\ \frac{m}{s}$

For the next 5.4 m, d=5.9 mm

$d_{23} := .0059\ m$ $\quad A_{23} := \frac{\pi}{4} \cdot d_{23}^{\,2}$ $\quad A_{23} = 2.734 \cdot 10^{-5}\ m^2$ $\quad G_{23} := \frac{mdot \cdot 2}{A_{23}}$ $\quad G_{23} = 80.469\ \frac{kg}{m^2 \cdot s}$ $\quad L_{23} := 5.4\ m$

$R_{23} := \frac{G_{23} \cdot d_{23}}{\mu_g}$ $\quad R_{23} = 4.592 \cdot 10^4$ $\quad f_{23} := 0.079 \cdot R_{23}^{\,-0.25}$ $\quad f_{23} = 5.397 \cdot 10^{-3}$ $\quad p_2 = 4.137 \cdot 10^4\ Pa$

$G_{23}^{\,2} \cdot R \cdot T_r \cdot \left(\frac{4 \cdot f_{23} \cdot L_{23}}{d_{23}} \right) = 1.128 \cdot 10^9\ \frac{kg^2}{m^2 \cdot s^4}$ $\quad p_2^{\,2} = 1.712 \cdot 10^9\ \frac{kg^2}{m^2 \cdot s^4}$

$p_3 := \left[p_2^{\,2} - G_{23}^{\,2} \cdot R \cdot T_r \cdot \left(\frac{4 \cdot f_{23} \cdot L_{23}}{d_{23}} \right) \right]^{\frac{1}{2}}$ $p_3 = 2.415 \cdot 10^4 \cdot Pa$ $p_{3mbar} := \frac{p_3}{1 \cdot 10^5\ Pa} \cdot 1000$ $p_{3mbar} = 241.505$

$\Delta P_{23} := p_2 - p_3$ $\Delta P_{23} = 1.722 \cdot 10^4\ Pa$ $\rho_3 := \frac{p_3}{R \cdot T_r}$ $\rho_3 = 2.738\ \frac{kg}{m^3}$ $Q_3 := \frac{mdot \cdot 2}{\rho_3}$

$Q_3 = 2.892 \cdot \frac{m^3}{hr}$ one Edwards compressor will handle 2.77 stave groups(5.5 staves)

$u_3 := \frac{Q_3}{A_{23}}$ $u_3 = 29.386\ \frac{m}{s}$ Mach. number ~0.3 $\frac{\rho_3 \cdot u_3^{\,2}}{1 \cdot 10^5\ Pa} \cdot 1000 = 23.647$ mbar, indication of elbow loss magnitude

For the next 25 m, d=13 mm

$d_{34} := .013\ m$ $A_{34} := \frac{\pi}{4} \cdot d_{34}^{\,2}$ $A_{34} = 1.327 \cdot 10^{-4}\ m^2$ $G_{34} := \frac{mdot \cdot 2}{A_{34}}$ $G_{34} = 16.575\ \frac{kg}{m^2 \cdot s}$ $L_{34} := 25\ m$

$R_{34} := \frac{G_{34} \cdot d_{34}}{\mu_g}$ $R_{34} = 2.084 \cdot 10^4$ $f_{34} := 0.079 \cdot R_{34}^{\,-0.25}$ $f_{34} = 6.575 \cdot 10^{-3}$ $p_3 = 2.415 \cdot 10^4\ Pa$

$G_{34}^{\,2} \cdot R \cdot T_r \cdot \left(\frac{4 \cdot f_{34} \cdot L_{34}}{d_{34}} \right) = 1.225 \cdot 10^8\ \frac{kg^2}{m^2 \cdot s^4}$ $p_3^{\,2} = 5.832 \cdot 10^8\ \frac{kg^2}{m^2 \cdot s^4}$

$p_4 := \left[p_3^{\,2} - G_{34}^{\,2} \cdot R \cdot T_r \cdot \left(\frac{4 \cdot f_{34} \cdot L_{34}}{d_{34}} \right) \right]^{\frac{1}{2}}$ $p_4 = 2.146 \cdot 10^4 \cdot Pa$ $p_{4mbar} := \frac{p_4}{1 \cdot 10^5\ Pa} \cdot 1000$ $p_{4mbar} = 214.639$

$\Delta P_{34} := p_3 - p_4$ $\Delta P_{34} = 2.687 \cdot 10^3\ Pa$ $\rho_4 := \frac{p_4}{R \cdot T_r}$ $\rho_4 = 2.434\ \frac{kg}{m^3}$ $Q_4 := \frac{mdot \cdot 2}{\rho_4}$

$Q_4 = 3.254 \cdot \frac{m^3}{hr}$ one Edwards compressor will handle 1.8 stave groups (3.4 staves)

$u_4 := \frac{Q_4}{A_{34}}$ $u_4 = 6.811\ \frac{m}{s}$

Limit on tube length of 4.7 mm ID $d := .0047\ m$ $mdot = 1.1\cdot 10^{-3}\ \frac{kg}{s}$ $\mu_g = 1.034\cdot 10^{-5}\ \frac{kg}{m\cdot s}$

$A := \frac{\pi}{4}\cdot d^2$ $G := \frac{mdot\cdot 2}{A}$ $G = 126.805\ \frac{kg}{m^2\cdot s}$ $G^2 = 1.608\cdot 10^4\ \frac{kg^2}{m^4\cdot s^2}$

$R = 34.929\ \frac{m^2}{s^2\cdot K}$ $T := (273 + T)\ K$ $T = 252.5\ K$ $f := 0.079\cdot\left(\frac{G\cdot d}{\mu_g}\right)^{-0.25}$ $f = 5.099\cdot 10^{-3}$

$p_{sexit} = 4.5\cdot 10^4\ Pa$ $\rho_g := \frac{p_{sexit}}{R\cdot T}$ assuming no pressure loss or recovery going from two staves into one

$u_g := \frac{G}{\rho_g}$ $u_g = 24.852\ \frac{m}{s}$

With reference to conversation with Greg sherwood of 3M, he indicated that properties of refrigerant R114 are similar to C_4F_{10} . We wish only to use this point to establish an approximate specific heat ratio and sonic speed in the vapor.

$R = 34.929\cdot\frac{J}{kg\cdot K}$

$c_p := 650\ \frac{J}{kg\cdot K}$ $k := \frac{c_p}{c_p + R}$ $k = 0.949$

Or if we estimate the specific heat constant pressure from the enthalpy tables for C_4F_{10} we get

$c_p := 704\ \frac{J}{kg\cdot K}$ $k := \frac{c_p}{c_p + R}$ $k = 0.953$

Let $k := 1.4$

$M_c := \left(\frac{1}{k}\right)^{\frac{1}{2}}$ $M_c = 0.845$ the critical Mach number for specific heat ratio of 0.913, which must be less than 1. Clearly we do not have the right specific ratio heat yet

$u_{critical} := (k\cdot R\cdot T)^{\frac{1}{2}}$ $u_{critical} = 111.118\ \frac{m}{s}$ which is close the anticipated value given by Greg Hallewell

$M := \frac{u_g}{u_{critical}}$ $L_{max} := \frac{d}{4\cdot f}\cdot\left(\frac{1 - k\cdot M^2}{k\cdot M^2} + \ln\left(k\cdot M^2\right)\right)$ $L_{max} = 2.448\ m$ at this point the duct length can not be increased further. the proposed length at this diameter was 1.5 m

$\left(\frac{M}{M_c}\right)^2 = 0.07$ $1 - k\cdot M^2\cdot\left(2\cdot\ln\left(\frac{M_c}{M}\right) + \frac{4\ f\cdot L_{max}}{d}\right) = 0.07$ it checks

The flow exit velocity after 2.5 meters is given by $u_{critcal}$, that is it is flow limited. The presure at this exit point is given by:

$$p_x := p_{sexit} \cdot \frac{u_g}{u_{critical}} \qquad p_x = 1.006 \cdot 10^4 \text{ Pa} \qquad \frac{p_x \cdot 1000}{1 \cdot 10^5 \text{ Pa}} = 100.645 \text{ mbar}$$

It is believed that if you connect several staves together and run the outlet to the compressor that one can test for this flow limit.

One may note that the equation for L_{max} is really independent of the value of k chosen since kM^2 has k in the numerator and denominator. Thus, our lack of information on the specific heat ratio is not disrupting our conclusion.

Lets choose another starting diameter

Limit on tube length of 5.9 mm ID $\quad d := .0059 \text{ m} \qquad mdot = 1.1 \cdot 10^{-3} \frac{kg}{s} \qquad \mu_g = 1.034 \cdot 10^{-5} \frac{kg}{m \cdot s}$

$$A := \frac{\pi}{4} \cdot d^2 \qquad G := \frac{mdot \cdot 2}{A} \qquad G = 80.469 \frac{kg}{m^2 \cdot s} \qquad G^2 = 6.475 \cdot 10^3 \frac{kg^2}{m^4 \cdot s^2}$$

$$R = 34.929 \frac{m^2}{s^2 \cdot K} \qquad T := (273 + T) \text{ K} \qquad T = 252.5 \text{ K} \qquad f := 0.079 \cdot \left(\frac{G \cdot d}{\mu_g}\right)^{-0.25} \quad f = 5.397 \cdot 10^{-3}$$

$$p_{sexit} = 4.5 \cdot 10^4 \text{ Pa} \qquad \rho_g := \frac{p_{sexit}}{R \cdot T}$$

assuming no pressure loss or recovery going from two staves into one

$$u_g := \frac{G}{\rho_g} \qquad u_g = 15.771 \frac{m}{s}$$

Let k=1.2 $\quad k := 1.2$

$$M_c := \left(\frac{1}{k}\right)^{\frac{1}{2}} \qquad M_c = 0.913$$

the critical Mach number for specific heat ratio of 0.913, which must be less than 1. Clearly we do not have the right specific ratio heat yet

$$u_{critical} := (k \cdot R \cdot T)^{\frac{1}{2}} \qquad u_{critical} = 102.875 \frac{m}{s}$$

which is close the anticipated value given by Greg Hallewell

$$M := \frac{u_g}{u_{critical}} \qquad L_{max} := \frac{d}{4 \cdot f} \cdot \left(\frac{1 - k \cdot M^2}{k \cdot M^2} + \ln\left(k \cdot M^2\right)\right) \qquad L_{max} = 8.443 \text{ m}$$

at this point the duct length can not be increased further. the proposed length at this diameter was 1.5 m

$$\left(\frac{M}{M_c}\right)^2 = 0.028 \qquad 1 - k \cdot M^2 \cdot \left(2 \cdot \ln\left(\frac{M_c}{M}\right) + \frac{4 \cdot f \cdot L_{max}}{d}\right) = 0.028$$

it checks

$p_x := p_{sexit} \cdot \frac{u_g}{u_{critical}}$ $\quad p_x = 6.899 \cdot 10^3$ Pa $\quad \frac{p_x \cdot 1000}{1 \cdot 10^5 \text{ Pa}} = 68.986$ mbar

the pressure continues to drop if we extend to this limit, however, we are proposing to limit the duct length to 1.5 meters.

At the distance of 1.5 meter the pressure is given by the earlier solution on page 4 of this appendix

$p_2 = 4.137 \cdot 10^4$ Pa $\quad \frac{p_2 \cdot 1000}{1 \cdot 10^5 \text{ Pa}} = 413.712$ mbar, and this is a clear improvement.

APPENDIX B

Calculation of Fluid Quality for C_4F_{10} +15.8 Deg C Inlet

Liquid properties at injector inlet

T	Pl	Tl	H1	Liquid state	
C	bar	K	KJ/mol		
15.000	1.910	288.000	21.029		
15.846	***1.965***	*288.846*	***21.246***	*0.423*	Interpolation factor
17.000	2.040	290.000	21.543		

Saturated vapor conditions in stave

T	T4m	P4m	H4m	X4	quality after injection
C	K	bar	KJ/mol		
-20.488	252.512	***0.450***	21.246	0.375	
-18.250	254.750	0.500	21.246	0.351	
-15.000	258.000	***0.580***	21.246	0.321	Fluid Inlet quality if Δ P = 130 mbar in stave

Saturated Fluid Properties of C4F10, used to find quality at injector exit

T	Pvp	Tvp	Sl	ΔSvap	Sv	Hl	ΔHvap	Hv	Interpolation	
C	bar	K	J/mol-K	J/mol-K	J/mol-K	KJ/mol	KJ/mol	KJ/mol	factor	
-21.000	0.439	252.000	53.450	96.960	150.400	12.050	24.520	36.570		
-20.488	***0.450***	*252.512*	*53.939*	*96.960*	*150.400*	*12.050*	*24.520*	*36.570*	*0.255814*	**Listed as stave exit condition**
-19.000	0.482	254.000	55.360	95.610	150.970	12.540	24.370	36.910		
-18.250	0.500	*254.750*	*56.076*	*95.111*	*151.184*	*12.720*	*24.314*	*37.034*	*0.375*	**Entering Condition?**
-17.000	0.530	256.000	57.270	94.280	151.540	13.020	24.220	37.240		
-15.000	***0.580***	258.000	59.170	92.960	152.130	13.510	24.070	37.580		
-13.000	0.635	260.000	61.050	91.660	152.720	14.000	23.920	37.920		

Minimum mass flow for dry vapor at exit Heat absorbed 72 watts(72J/sec)

(assumes entering pressure at .58 bar and exit at 0.45 bar)

H4m	H1e	ΔHmax	mdot	mdot	Molecular weight 238 g/g-mol
KJ/mol	KJ/mol	KJ/mol	mol/sec	g/sec	
21.246	36.570	15.324	0.005	1.118	

For mass flow of 0.7cc/sec at 1.52 g/cc density

mdot	Agreement within
g/sec	%
1.094	2.182

The fluid must be dry leaving the stave

Looking at 2.5 bar inlet and same outlet conditions

Liquid properties at injector inlet

Pl	Tl	H1
bar	K	KJ/mol
2.500	296.000	23.097

Saturated vapor conditions in stave

T	T4m	P4m	H4m	X4	
C	K	bar	KJ/mol		
-15.000	258.000	***0.580***	23.097	0.398	this increases the quality and potential for dry-out

Exit fluid property data

T	Pvp	Tvp	Rho_l	Rho_v	Interpolation	
C	bar	K	kg/m^3	kg/m^3	factor	
-21.000	0.439	252.000	1675.80	5.140		
-20.488	***0.450***	*252.512*	*1674.14*	*5.140*	*0.255814*	**stave exit**
-19.000	0.482	254.000	1669.30	5.620		
-18.250	*0.500*	*254.750*	*1666.15*	*5.811*	*0.375*	
-17.000	0.530	256.000	1660.90	6.130		
-15.000	***0.580***	258.000	1653.40	6.680		**injector exit**
-13.000	0.635	260.000	1646.00	7.270		

APPENDIX C

Calculation of Fluid Quality for C_4F_{10}-at -5 Deg. C

Liquid properties at injector inlet

T	Pl	Tl	Hl		
C	bar	K	KJ/mol	Liquid state **point 3**	
-5.000	1.000	268.000	15.964		
-5.000	***1.965***	*268.000*	***15.954***	*0.965*	Interpolation factor
-5.000	2.000	268.000	15.954		

Saturated vapor conditions in stave

T	T4m	P4m	H4m	X4	
C	K	bar	KJ/mol		quality after injection
-20.49	252.512	***0.450***	15.954	0.159	
-18.25	254.750	0.500	15.954	0.133	
-17.00	256.000	***0.530***	15.954	0.102	Fluid inlet quality assuming the ΔP=78mbar, **point 4**

Saturated Fluid Properties of C4F10, used to find quality at injector exit

T	Pvp	Tvp	Sl	ΔSvap	Sv	Hl	ΔHvap	Hv	Interpolation factor	
C	bar	K	J/mol-K	J/mol-K	J/mol-K	KJ/mol	KJ/mol	KJ/mol		
-21.000	0.439	252.000	53.450	96.960	150.400	12.050	24.520	36.570		
-20.488	***0.450***	*252.512*	*53.939*	*96.960*	*150.400*	*12.050*	*24.520*	*36.570*	*0.255814*	**Listed as stave exit condition**
-19.000	0.482	254.000	55.360	95.610	150.970	12.540	24.370	36.910		
-18.250	0.500	*254.750*	*56.076*	*95.111*	*151.184*	*12.720*	*24.314*	*37.034*	*0.375*	
-17.000	***0.530***	256.000	57.270	94.280	151.540	13.020	24.220	37.240		**Entering Condition**
-15.000	0.580	258.000	59.170	92.960	152.130	13.510	24.070	37.580		
-13.000	0.635	260.000	61.050	91.660	152.720	14.000	23.920	37.920		

Minimum mass flow for dry vapor at exit Heat absorbed 72 watts(72J/sec)

(assumes entering pressure at .53 bar and exit at 0.45 bar)

H4m	H1e	ΔHmax		mdot	mdot	Molecular weight 238 g/g-mol
KJ/mol	KJ/mol	KJ/mol		mol/sec	g/sec	
15.954	36.570	20.616		0.0035	0.831	minimum flow for to pick up 72 watts

24

15.65217

Hl	ΔHvap	ΔHreq	x_1	**point 1**		
KJ/mol	KJ/mol	KJ/mol				
12.050	24.520	15.66	0.80	0.0046	1.094	actual flow

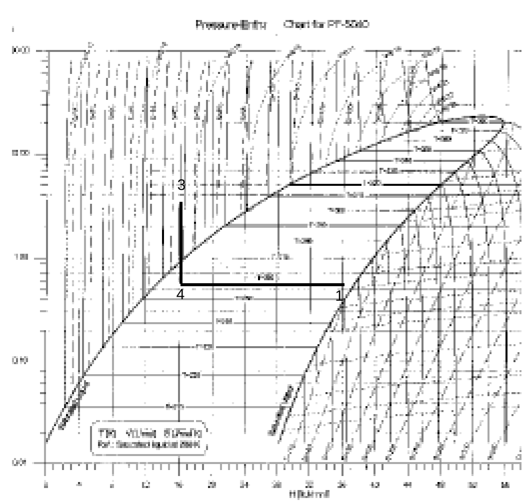
For mass flow of 0.7cc/sec at 1.52 g/cc density

mdot
g/sec
1.094

The fluid most likely has a quality of 0.8 leaving the stave. Takes another 23 watts to dry out the fluid.

Fluid density data

T	Pvp	Tvp	Rho_l	Rho_v	Interpolation factor	
C	bar	K	kg/m^3	kg/m^3		
-21.000	0.439	252.000	1675.80	5.140		
-20.488	***0.450***	*252.512*	*1674.14*	*5.140*	*0.255814*	**stave exit**
-19.000	0.482	254.000	1669.30	5.620		
-18.250	0.500	*254.750*	*1666.15*	*5.811*	*0.375*	
-17.000	***0.530***	256.000	1660.90	6.130		**Injector exit**
-15.000	0.580	258.000	1653.40	6.680		
-13.000	0.635	260.000	1646.00	7.270		



System flow analysis-Stave Barrel C4F10, -5 Deg. C inlet to injector, Quality of 10% after injection

1.0 Pressure Drop Calculations In Stave

Pressure Drop Calculations In Stave

Frictional Pressure Drop-Equation 1

$\nu_{liq} := 0.415 \cdot 10^{-6} \frac{m^2}{s}$ at -17 deg C, after injection (different from appendix A by pressure drop effect down stave)

$\rho_{liq} := 1660.9 \cdot \frac{kg}{m^3}$ $\rho_g := 6.13 \cdot \frac{kg}{m^3}$ $x_i := 0.1$ $x_o := .8$ $\mu_{liq} := \rho_{liq} \cdot \nu_{liq}$ $\mu_{liq} = 6.893 \cdot 10^{-4} \cdot Pa \cdot s$

$mdot := 1.1 \cdot 10^{-3} \frac{kg}{s}$ $d := 0.0034 \cdot m$ $A := \frac{\pi}{4} \cdot d^2$ $A = 9.079 \cdot 10^{-6}\, m^2$

$G := \frac{mdot}{A}$ $G = 121.156 \frac{kg}{m^2 \cdot s}$ $G^2 = 1.468 \cdot 10^4 \frac{kg^2}{m^4 \cdot s^2}$

$R_{liq} := \frac{G \cdot d}{\mu_{liq}}$ $R_{liq} = 597.63$ $X_o := x_o \cdot \left(\frac{\rho_{liq} - \rho_g}{\rho_g} \right)$ $X_i := x_i \cdot \left(\frac{\rho_{liq} - \rho_g}{\rho_g} \right)$

$C_{flo} := 0.079 \cdot R_{liq}^{-0.25}$ $C_{flo} = 0.016$ $L := 0.8\, m$

$\Delta P_F := \frac{2 \cdot L}{d} \cdot C_{flo} \cdot \frac{G^2}{\rho_{liq}} \cdot \left(1 + \frac{X_o + X_i}{2} \right)$ $\Delta P_F = 8.139 \cdot 10^3\, Pa$ $\Delta P_F = 61.046 \cdot torr$

$\Delta Pmbar_F := \frac{\Delta P_F \cdot 1000}{1 \cdot 10^5\, Pa}$ $\Delta Pmbar_F = 81.387$

Gravitational Pressure Drop

This pressure drop contribution is zero because the stave position remains horizontal

Accelerational Pressure Drop-Equation 3

$\Delta P_A := \frac{G^2}{\rho_{liq}} \left(X_o - X_i \right)$ $\Delta P_A = 1.67 \cdot 10^3\, Pa$ $\Delta Pmbar_A := \frac{\Delta P_A \cdot 1000}{1 \cdot 10^5\, Pa}$ $\Delta Pmbar_A = 16.7$

Combined Pressure Drop

$\Delta P := \Delta P_F + \Delta P_A$ $\Delta P = 9.809 \cdot 10^3\, Pa$ $\Delta Pmbar := \Delta Pmbar_F + \Delta Pmbar_A$ $\Delta Pmbar = 98.088$

Since the pressure decays this amount we can expect the exit temperature to drop by ~4.5 Deg. C(Note: ATLAS Cooling group meeting notes of 6 Nov. 1998 gave stave tube surface temperatues of -11.5 entrance, -12.3 mid, and -13.4 exit for an overall change of 2 Deg. C. This is not necessarily an exact indication of the inside gas temperatures but it does provide a basis for comparison with predictions.

Flow State Considerations

$T := -17$ gas viscosity temperature in degree C

$\mu_g := 1.1165\cdot 10^{-5}\ Pa\cdot s + \left(4.0254\cdot 10^{-8}\ T\right)\ Pa\cdot s \quad \mu_g = 1.048\cdot 10^{-5}\ \frac{kg}{m\cdot s}$

$\frac{\mu_g}{\mu_{liq}} = 0.015 \quad \frac{\rho_g}{\rho_{liq}} = 3.691\cdot 10^{-3}$

Void fraction at entrance and exit of stave

$G_{gin} := x_i\cdot G \quad V_{gin} := \frac{G_{gin}}{\rho_g} \quad V_{gin} = 1.976\ \frac{m}{s} \quad G_{gout} := x_o\cdot G \quad V_{gout} := \frac{G_{gout}}{\rho_g} \quad V_{gout} = 15.812\ \frac{m}{s}$

$G_{lin} := \left(1 - x_i\right)\cdot G \quad V_{lin} := \frac{G_{lin}}{\rho_{liq}} \quad V_{lin} = 0.066\ \frac{m}{s} \quad G_{lout} := \left(1 - x_o\right)\cdot G \quad V_{lout} := \frac{G_{lout}}{\rho_{liq}} \quad V_{lout} = 0.015\ \frac{m}{s}$

$\alpha_i := \frac{1}{1 + \left(\frac{V_{gin}}{V_{lin}}\cdot\frac{1 - x_i}{x_i}\cdot\frac{\rho_g}{\rho_{liq}}\right)} \quad \alpha_o := \frac{1}{1 + \left(\frac{V_{gout}}{V_{lout}}\cdot\frac{1 - x_o}{x_o}\cdot\frac{\rho_g}{\rho_{liq}}\right)}$ must be based on actual velocities, here we are using superficial velocities

$\alpha_i = 0.5 \quad \alpha_o = 0.5$ taken as an estimate

$u_{gin} := \frac{V_{gin}}{\alpha_i} \quad u_{gin} = 3.953\ \frac{m}{s} \quad u_{lin} := \frac{V_{lin}}{1 - \alpha_i} \quad u_{lin} = 0.131\ \frac{m}{s}$ first cut at actual velocities

$\alpha_{icorr} := \frac{1}{1 + \left(\frac{u_{gin}}{u_{lin}}\cdot\frac{1 - x_i}{x_i}\cdot\frac{\rho_g}{\rho_{liq}}\right)} \quad \alpha_{icorr} = 0.5$ void fraction did not change, based on first cut at actual velocities

$u_{gout} := \frac{V_{gout}}{\alpha_o} \quad u_{gout} = 31.623\ \frac{m}{s} \quad u_{lout} := \frac{V_{lout}}{1 - \alpha_o} \quad u_{lout} = 0.029\ \frac{m}{s} \quad \frac{1 - x_o}{x_o} = 0.25 \quad \frac{\rho_g}{\rho_{liq}} = 3.691\cdot 10^{-3}$

$\frac{u_{gout}}{u_{lout}} = 1.084\cdot 10^{3}$

$\alpha_{ocorr} := \frac{1}{1 + \left(\frac{u_{gout}}{u_{lout}}\cdot\frac{1 - x_o}{x_o}\cdot\frac{\rho_g}{\rho_{liq}}\right)} \quad \alpha_{ocorr} = 0.5$ again the void fraction did not change

$\alpha_{hin} := \frac{1}{1 + \frac{1 - x_i}{x_i}\cdot\frac{\rho_g}{\rho_{liq}}} \quad \alpha_{hin} = 0.968 \quad \alpha_{hout} := \frac{1}{1 + \frac{1 - x_o}{x_o}\cdot\frac{\rho_g}{\rho_{liq}}} \quad \alpha_{hout} = 0.999$ homogeneous void fraction is noticeably higher

Flow Parameters in stave

$\rho_{air} := 1.23 \frac{kg}{m^3}$ $\rho_{water} := 1000 \frac{kg}{m^3}$ $\psi := \left(\frac{\rho_g}{\rho_{air}} \cdot \frac{\rho_{liq}}{\rho_{water}}\right)$

$\frac{G_{gin}}{\psi} = 1.464 \frac{kg}{m^2 \cdot s}$ $\psi \cdot G_{lin} = 902.581 \frac{kg}{m^2 \cdot s}$ $\frac{G_{gout}}{\psi} = 11.709 \frac{kg}{m^2 \cdot s}$ $\psi \cdot G_{lout} = 200.574 \frac{kg}{m^2 \cdot s}$

slug flow in annular flow out